

Circular Model on Forecasting Returns of Sri Lankan Share Market

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Abstract: Forecasting is essential for a healthy stock market. Mathematical and statistical modeling plays a vital role in forecasting share returns. A statistical model named as Capital Asset Pricing Model (CAPM) is the mostly used model for the purpose all over the world. Literature gives enough evidence for the inefficiency of CAPM in forecasting returns. Yet, Sri Lankan stock market depends on CAPM for forecasting risk and returns of securities. Literature revealed that the Sri Lankan share market returns follow wave like patterns with no trend. This study tested a new forecasting model for forecasting individual company returns. The model, which is named as “Circular Model”, is based on Fourier transformation.

Keywords: Mathematical models, Statistical models, Fourier transformation, Circular Model.

I. INTRODUCTION

Predicting the future was one of the strongest desires of a man, which may have begun with the beginning of time. Predictions were immense important to the man at individual levels as well as in general. The actualization of predictions, historically begun with fortune telling, astrology and palmistry etc., paved the path for scientific predictions and forecasting. Scientific forecasting is based on mathematical modeling.

A mathematical model is a simplification of a real world situation into an equation or a set of equations. Process of designing a mathematical model split into several stages. They are; a real world problem is observed, formed hypotheses, a mathematical model is devised, experimental data is collected from the real world, the mathematical model is used to predict the expected behavior of the real world problem, compare predicted and observed outcomes and refined the mathematical model (if necessary).

Classification of Mathematical Models:

Mathematical models can be classified in many ways. Some of them are:

- i. Empirical and Mechanistic models.
- ii. Static and Dynamic models.
- iii. Deterministic and Stochastic models.

All mathematical models have model parameters. In an empirical model, parameters do not have any biological interpretation, but in mechanistic models, parameters have specific biological interpretation. Therefore empirical models are known as “Black- Box” models and “mechanistic models are known as “White- Box” models. A model is said to be

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“Static” when it does not have time- dependent component. In contrast, dynamic models contain time-dependent component. A deterministic model is one in which every set of variable states is uniquely determined by parameters in the model and by sets of previous states of these variables. Deterministic models are not associated with any randomness. Conversely, in a stochastic model, randomness is present and variable states are described by associated probability distributions. In general stochastic models are referred as Statistical models.

Stock Market Forecasting:

Risk and return are most important concepts in financial markets (Pande, 2004). Investors expect higher returns at a lower risk; as such, they are very much concerned about the information on the risk and return of individual assets. Therefore, forecasting risk and return of assets were of immense interest over the past decades.

Statistical techniques and soft computing techniques are the two main strands of stock market forecasting. Statistical techniques comprise of Fundamental analysis and Technical analysis. Fundamental analysis involves analyzing the economic factors or characteristics of a company, namely; company value, company earnings, book-to-market equity etc. On the other hand, the interest in the technical analysis is price movements and trading volume in the market.

Modern Portfolio theory of Markowitz (1952) was one milestone of fundamental analysis. Tobin (1958), Treynor (1961), Sharpe (1964), Lintner (1965), Mossin (1966), Black (1972) and many others contributed to for the development of Markowitz (1952), and their combined output is known as the Capital Asset Pricing Model (CAPM). It is given by the formula;

$$E(R_i) = R_f + \beta(R_m - R_f) \quad (1)$$

Where R_i is the return of i^{th} company assets, R_m is the return of total market, R_f is the risk free rate of return and

$\beta = \frac{Cov(R_i, R_m)}{\sigma_m^2}$. The β coefficient, which is known as the risk factor is the key parameter in CAPM. It is considered

that, if $\beta = 0$, share price is not at all correlated with the market, therefore no risk. If $\beta = +1$, an average level of risk. If $\beta > 1$, security returns fluctuates more than the market returns, therefore high risk. If $\beta < 1$, asset inversely follows the market. CAPM has been widely used in stock market forecasting all over the world.

Share trading in Sri Lanka:

Share trading in Sri Lanka commenced in the 19th century, with the formation of Colombo Share Brokers Association. Later it was renamed as ‘Colombo Stock Exchange (CSE). According to official website of CSE, 294 companies were listed for year 2014, representing twenty business sectors. Fundamental Analysis approach of asset pricing CAPM has been used for Sri Lankan share market forecasting. β coefficients for listed companies are published by CSE on quarterly basis and use by the investors in their investment decisions.

PROBLEM STATEMENT:

CAPM model has been subjected to extensive empirical testing in the past few decades. The central assertion of the CAPM is that, there exist a linear relationship between the expected return and market risk (β). This was first argued by Banz (1981); Black, Jensen and Scholes (1972); Fama and MacBeth (1973) and many others. Their findings have given considerable evidence that risk itself cannot explain returns of individual securities and hence the portfolio returns. Also literature gives evidence for incapability of CAPM in Sri Lankan share market. Yet the Sri Lankan stock market still depends on CAPM.

Literature revealed that Konarasinghe & Pathirawasam (2013); Konarasinghe, Abeynayake, & Gunaratne (2016), Konarasinghe, Abeynayake, & Gunaratne (2015), Rathnayaka, Seneviratna, & Nagahawatta (2014) and many others have attempted to establish a suitable forecasting technique for the Sri Lankan share market. But forecasting ability of suggested methods were not satisfying, thus it is essential to develop suitable forecasting techniques for the Sri Lankan share market.

II. METHODOLOGY

Listed companies of Colombo Stock Exchange (CSE) in year 2014 were the population of the study. The population consists of 294 companies in 20 business sectors. Daily closing share prices of individual companies, monthly indices of business sectors and All Share Price Indices (ASPI) from year 1991 to year 2014 were obtained from CSE data library 2014. A random sample of ten companies was selected for analysis.

Monthly average share prices for individual companies were calculated by daily closing share prices. One standard formula for calculating share return at time t is;

$$R_t = \left(\frac{P_t - P_{t-1}}{P_{t-1}} \right) 100 \quad (2)$$

Where; P_t is the share price at time t (Pande, 2005).

Outlier Adjustment

Outliers are the extremely large or small values outside the overall pattern of a data set. Outlier detection and adjustment are essential in data analysis. Boundaries of outliers are defined in many ways. Following rule is often used in outlier detection (Attwood, Clegg, Dyer & Dyer, 2008).

$$\begin{aligned} L &= Q_1 - 1.5 * IQR \\ U &= Q_3 + 1.5 * IQR \end{aligned} \quad (3)$$

Where Q_1 , Q_3 are the lower quartile and upper quartile respectively, IQR is the inter quartile range, L is the lower boundary and U is the upper boundary. Any data value above U or below L was considered as outliers. Such data points were adjusted by taking moving average of order three, using a computer program written in MATLAB.

Accuracy of this program is based on two assumptions;

- i. First three values of the array are not being outliers.
- ii. Three consecutive outliers not being occurred.

Outliers were manually adjusted, when one or two of the assumptions were violated.

Mathematical and Statistical Methods Used in the Study:

Forecasting model suggested in the study is named as "Circular Model (CM). It combines Fourier transformation and multiple regression analysis.

Fourier Transformation:

Fourier transformation is a linear transformation. A transformation moves all the points in (x, y) plane according to some rule. A special property of linear transformation is that, it involves only linear expressions of x and y (Attwood, Cope, Moran, Pateman, Pledger, Staley & Wilkins, 2008). Rotations, reflections and enlargements are examples for linear transformation. Fourier transformation employs rotation. Fourier transformation (FT) can be used to transform a real valued function $f(x)$ into series of trigonometric functions (Philippe, 2008). FT has two versions; discrete transformation and continuous transformation. Discrete version of Fourier transformation is;

$$f(x) = \sum_{-\infty}^{\infty} a_n e^{-k\theta} \quad (4)$$

According to De Moivre's theorem;

$$e^{-k\theta} = \cos k\theta + i \sin k\theta \quad (5)$$

Where, i is a complex number. Therefore $f(x)$ can be written as:

$$f_x = \sum_{k=1}^n a_k \cos k\theta + b_k \sin k\theta \quad (6)$$

Where a_k and b_k are amplitudes; k is the harmonic of oscillation. Highest harmonic (k) is defined as (Stephen, 1998);

$$k = \begin{cases} n/2: & n \text{ even} \\ (n-1)/2: & n \text{ odd} \end{cases}$$

Fourier transformation is in cooperated to a uniform circular motion of a particle in a horizontal circle and basic trigonometric ratios. A particle P which is moving in a horizontal circle of centre O and radius a is given in Figure 1. V is the tangential speed of the particle and ω is the angular speed of the particle at time t . F is the centripetal force,

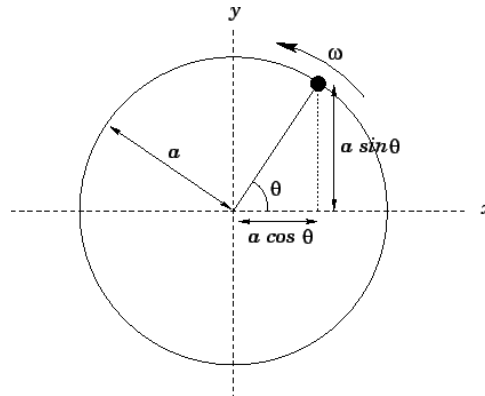


Figure 1: Motion of a particle in a horizontal circle

Angular speed (ω) is defined as the rate of change of the angle with respect to time. Then;

$$\theta = \omega t \tag{7}$$

At one complete circle $\theta=2\pi$ radians (360 degrees). Therefore, time taken for one complete circle (T) is given by:

$$T = 2\pi / \omega \tag{8}$$

Reference to Figure (1); $\vec{op} = a(\cos\theta\hat{i} + \sin\theta\hat{j})$, where, r is the amplitude or wave height. A wave with constant amplitude is defined as a regular wave and a wave with variable amplitude is known as the irregular wave.

Concept of Fourier transformation is applied in the present study for explaining returns. In circular motion, a time taken for one complete circle is known as the period of oscillation. In other words, period of oscillation is equal to the time between two peaks or troughs of sine or cosine function. If a time series follows a wave with f peaks in N observations, its period of oscillation can be given as;

$$T = \frac{\text{total number of periods}}{\text{total number of peaks}} = \frac{N}{f} \tag{9}$$

Equating (3-26) and (3-27); $\frac{2\pi}{\omega} = \frac{N}{f}$

Then $\omega = 2\pi \frac{f}{N}$ (10)

Hence the return at time t (R_t) was modeled as;

$$R_t = \sum_{k=1}^n (a_k \sin k\omega t + b_k \cos k\omega t) + \varepsilon_t \tag{11}$$

Number of observations per season (year) is $n=12$, hence $k=6$.

A program written in MATLAB was used for peak identification and ω calculation. Linear independence of trigonometric series $\sin(k\omega t)$ and $\cos(k\omega t)$ were confirmed by correlation analysis. Multiple regression technique was adopted for estimation of amplitudes a_k and b_k .

III. FINDINGS

Circular model was tested on random sample of ten companies of CSE. Goodness of fit tests and absolute measurements of errors were used in model validation. For example, Circular model was tested on returns of company EDEN as follows; Angular speed (ω) for company EDEN is calculated by;

$$\omega = 2\pi f / N$$

Where f is the number of peaks and N is the number of observations in the series. For EDEN, $f=45$ and $N=200$ for model fitting, hence $\omega=1.4137$. Then twelve trigonometric series; $\sin k\omega t$ and $\cos k\omega t$ for k is from 1 to 6 and $t=200$ were obtained. Correlation analysis confirmed the independence of these series. Hence R_t was regressed on them. Fitted Circular Model is;

$$R_t = -0.58047 + 1.8205\sin 4\omega t + 1.6831\cos 3\omega t - 2.1024\cos 4\omega t \tag{12}$$

Model for EDEN, given in (12) comprises three trigonometric functions; $\sin 4\omega t$, $\cos 3\omega t$ and $\cos 4\omega t$. In other words motion of returns comprises three circular motions; circle (c1) with angular speed $4\omega t$ and radius (r_1)1.8205, circle (c2) with angular speed $3\omega t$ and radius (r_2) 1.6831 and circle (c3) with angular speed $4\omega t$ and radius (r_3) 2.1024. Waves related to circular motions c1, c2, c3 are given in Figure 2, Figure 3, and Figure 4 respectively.

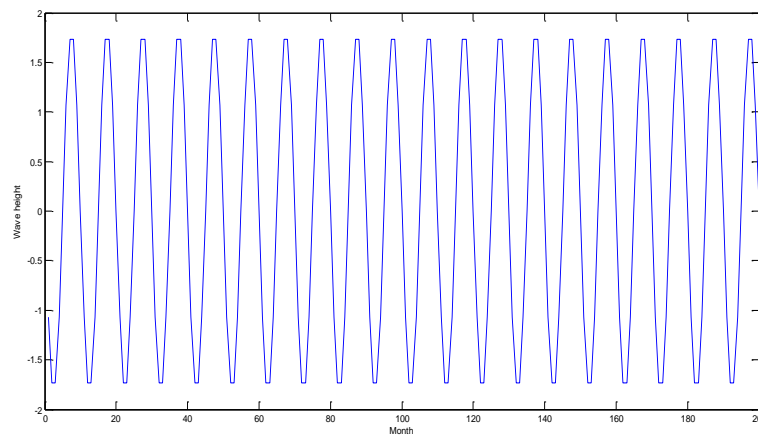


Figure 2: Plot of Circular Motion c1

Wave c1 is a regular wave with amplitude 1.8205 and period of the oscillation fifteen months.

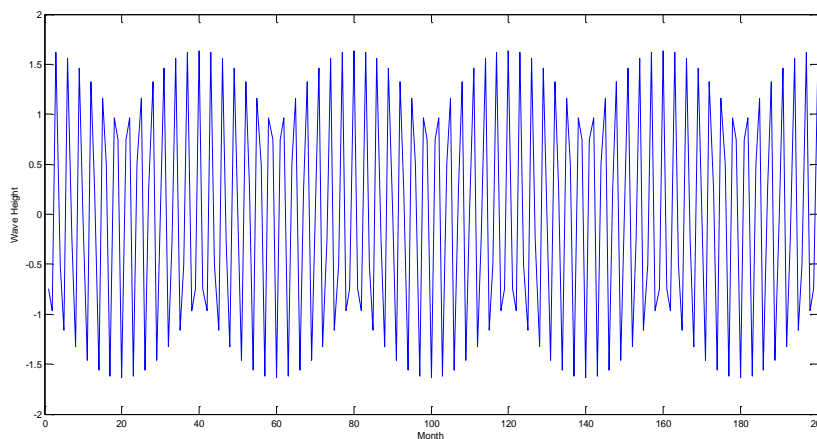


Figure 3: Plot of Circle c2

Wave c2 is an irregular wave with highest amplitude 1.6831 and period of the oscillation five months.

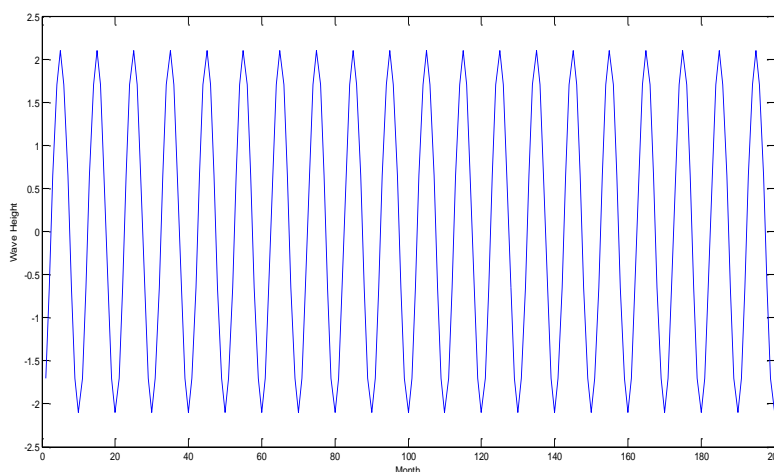


Figure 4: Plot of Circular Motionc3

Wave c3 is a regular wave with amplitude 2.1024 and period of the oscillation is sixteen months. Summation of waves related to circular motions c1, c2, c3;

$$(1.8205\sin 4\omega t + 1.6831\cos 3\omega t - 2.1024\cos 4\omega t)$$

R_t is a vertical translation of combined graphs by units 0.58047 downwards. It gives the fits for returns of EDEN.

RMSE of the model is 7.93 and MAD is 6.3 in model fitting and 5.7, 4.4 respectively in forecasting. Normal Probability Plot of Residuals and Histogram of Residuals suggested the normality of residuals, Anderson Darling test (P=0.581) confirmed it. Plot of Residual Vs Fitted Values did not show any pattern and they lie on both sides of zero. It suggests the independence of residuals. Modified Box-Pierce (Ljung-Box) Chi-Square statistic was greater than significance level (0.05), confirmed the independence of residuals.

Same procedure was repeated for the other companies and the summary of outputs is given in Table I. Circular model was fitted for eight out of the ten companies. RMSE and MAD of all the models were small in model fitting as well as forecasting. Assumptions of residuals; normality and independence were satisfied in fitted models. Therefore Circular model is suitable for forecasting returns of individual companies of Sri Lankan share market.

Table I: Fitted Circular Models for Individual Company returns

Company	Best Fitting Model	Model Fitting		Model Verification		Remarks of Residuals
		RMSE	MAD	RMSE	MAD	
EDEN	$R_t = -0.58047 + 1.8205\sin 4\omega t + 1.6831\cos 3\omega t - 2.1024\cos 4\omega t$	7.9	6.3	5.7	4.4	Normal, Uncorrelated
ABAN	$R_t = 1.9194 + 1.7358\cos 5\omega t$	5.7	4.5	6.7	5.2	Normal, Uncorrelated
LMF						Model does not fit
DFCC	$R_t = 0.31742 - 2.0337\cos 5\omega t$	8.3	6.5	6.06	4.96	Normal, Uncorrelated
AGAL	$R_t = -0.5995 + 2.1701\sin 3\omega t - 2.549.\sin 5\omega t$	8.8	7.1	7.8	5.9	Normal, Uncorrelated

NESTL	$R_t = 1.5412 - 1.4092\cos 3\omega t$	4.44	3.51	4.11	3.21	Normal, Uncorrelated
HAYL	Model does not fit					
CLAND	$R_t = -0.930 - 1.476\cos 6\omega t$	7.8	6.1	6.9	5.6	Normal, Uncorrelated
EQIT	$R_t = 0.5024 - 2.1354\sin \omega t$	7.9	6.4	9.1	7.2	Normal, Uncorrelated
DIMO	$R_t = -0.4712 - 1.3696\sin \omega t$	7.0	5.4	6.8	5.0	Normal, Uncorrelated
HASU	$R_t = 1.0822 - 2.1016\sin 5\omega t$	6.7	5.3	5.7	4.4	Normal, Uncorrelated

IV. CONCLUSION

Share trading is important to the investors, industries as well as the entire economy of a country. Share trading contribution to Sri Lankan Gross Domestic Product (GDP) has been increasing over the years. In year 2014, market capitalization of CSE was 2.4 trillion rupees and it corresponds to approximately 1/3 of the GDP (Wikipedia, 2016). It clearly shows the importance of share trading to the economy of the country.

Stock market investments are high risk investments. Thus, forecasting is the most important activity that helps to judge the market risk and grab scarce opportunities. Javad (1993) has pointed out the importance of an efficient pricing mechanism to a stock market. According to the author, "a major factor hindering the foreign investment in a market is lack of information about the price or return behavior of the market". CSE Annual Report (2014) revealed that domestic investor's attraction towards share trading has increased while foreign investor's attraction has decreased. This may be due to the inefficiency of the forecasting mechanism in Sri Lankan share market.

Some studies have shown that the risk factor (β) is not important at all in stock market forecasting. As such it is essential to develop an indicator to measure the risk of an investment. This study suggested a new forecasting model, named as Circular Model (CM). Results revealed that the CM was successful in forecasting monthly returns of 80% of the companies. It is concluded that CM is a suitable forecasting technique for Sri Lankan share market. But this technique is appropriate, only if the data series shows no trend.

It is recommended to test the CM on more companies of CSE for further confirmation.

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